101 $\int_{-1}^{2} x(1+x^3)\,dx$ 10

step 1: find antiderivative $\int x(1+x^3)\,dx$

$= \int x+x^4\,dx = \frac{x^2}{2}+\frac{x^5}{5}+C$

step 2: evaluate . . . .

$\int_{-1}^{2} x(1+x^3)\,dx = \left(\frac{x^2}{2}+\frac{x^5}{5}\right)\Big|_{-1}^{2}$

$= \left(\frac{2^2}{2}+\frac{2^5}{5}\right)-\left(\frac{(-1)^2}{2}+\frac{(-1)^5}{5}\right)$

$= \left(2+\frac{32}{5}\right)-\left(\frac{1}{2}-\frac{1}{5}\right) = \frac{33}{5}+\frac{3}{2} = \frac{81}{10}$

22) $\displaystyle\int_{-1}^{1} \frac{dx}{1+x^2}$

step 1 find antiderivative

$$\int \frac{1}{1+x^2}\,dx = \ldots = \ldots \text{ IDK?}$$

$\boxed{\text{p388} \quad \int \frac{1}{1+x^2}dx = \tan^{-1}x + C}$

$$\int \frac{1}{1+x^2}\,dx = \tan^{-1}(x) + C$$

step 2 eval

$$\int_{-1}^{1} \frac{1}{1+x^2}\,dx = \left(\tan^{-1} x\right)\Big|_{-1}^{1}$$

$$= \tan^{-1}(1) - \tan^{-1}(-1)$$

$$= \frac{\pi}{4} - \left(-\frac{\pi}{4}\right) = \frac{\pi}{2}$$

23)
$$\int_{\sqrt{2}}^{2} \frac{dx}{x\sqrt{x^2-1}}$$

step 1 +3sec

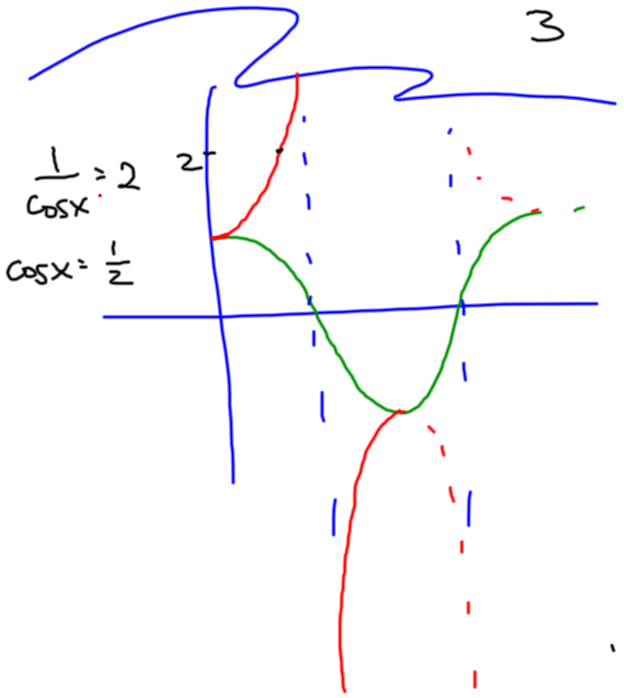
$$\int \frac{dx}{x\sqrt{x^2-1}} = \sec^{-1}|x| + C$$

step 2 eval

$$\int_{\sqrt{2}}^{2} \frac{dx}{x\sqrt{x^2-1}} = \sec^{-1}(x) \Big|_{\sqrt{2}}^{2}$$

$$= \sec^{-1}(2) - \sec^{-1}(\sqrt{2})$$

$$= \frac{\pi}{3} - \frac{\pi}{4} = \frac{\pi}{12}$$



what about $\sec^{-1}(x)$?

= "The angle θ whose secant is x"

= "The angle whose cosine is $\frac{1}{x}$"

$\therefore$ $\sec^{-1}(x) = \cos^{-1}\left(\frac{1}{x}\right)$

$\boxed{30}$ $\int_{-1}^{2} \sqrt{2+|x|}\,dx$

$= \int_{-1}^{0} \sqrt{2+(-x)}\,dx + \int_{0}^{2} \sqrt{2+x}\,dx$

$\int c\,dx = \int cx^0\,dx$

$\int \sqrt{2-x}\,dx$
$u = 2-x$
$du = -dx$
$-\int \sqrt{u}\,du = -\frac{u^{3/2}}{3/2}$
$= \frac{-2(2-x)^{3/2}}{3}$

$\int \sqrt{2+x}\,dx$
$u = x+2$
$du = dx$
$\int \sqrt{u}\,du = \frac{u^{3/2}}{3/2}$
$\frac{2}{3}(2+x)^{3/2}$

$\int_{-1}^{0} \sqrt{2-x}\,dx = \left(-\frac{2}{3}(2-x)^{3/2}\right)\Big|_{-1}^{0}$
$= -\frac{2}{3}(2-0)^{3/2} - \left(-\frac{2}{3}(2-(-1))^{3/2}\right)$
$= -\frac{2}{3}\left(2^{3/2} - 3^{3/2}\right)$

$\int_0^2 \sqrt{2+x}\,dx = \frac{2}{3}(2+x)^{3/2}\Big|_0^2$
$= \frac{2}{3}\left(4^{3/2}\right) - \frac{2}{3}\left(2^{3/2}\right)$

(b) $\int_0^{\pi/2} \left|\frac{1}{2} - \sin x\right| dx$ $\quad |-2| = -(-2)$ $\quad$ 30 53 45

$= \int_0^{\pi/6} \left|\frac{1}{2}-\sin x\right| dx + \int_{\pi/6}^{\pi/2} \left|\frac{1}{2}-\sin x\right| dx$ $\quad -(\frac{1}{2}-\sin x)$

$\frac{\pi}{6}$ $\frac{\pi}{2}$

$= \int_0^{\pi/6} \frac{1}{2} - \sin x\,dx + \int_{\frac{\pi}{6}}^{\pi/2} \sin x - \frac{1}{2}\,dx$

$= \left(\frac{1}{2}x - (-\cos x)\right)\Big|_0^{\pi/6} + \left(-\cos x - \frac{1}{2}x\right)\Big|_{\pi/6}^{\pi/2}$

$= \left(\frac{1}{2}\left(\frac{\pi}{6}\right) + \cos\frac{\pi}{6}\right) - (\cos 0) + \left(-\cos\frac{\pi}{2} - \frac{1}{2}\left(\frac{\pi}{2}\right)\right) - \left(-\cos\frac{\pi}{6} - \frac{1}{2}\left(\frac{\pi}{6}\right)\right)$

$= \frac{\pi}{6} + 2\cos\frac{\pi}{6} - 1 - \cos\frac{\pi}{2} - \frac{\pi}{4}$

$= \left(\frac{\pi}{6} - \frac{\pi}{4} - 1\right) + 2\left(\frac{\sqrt{3}}{2}\right) - (0)$

$= -\left(1 + \frac{\pi}{12}\right) + \sqrt{3}$

$|f(x)|$ will chg when $f(x) = 0$

Note: Whenever you encounter $|f(x)|$ consider subdividing the x-interval into pcs on which the f^n is 0 or positive & pcs on which the f^n is negative!

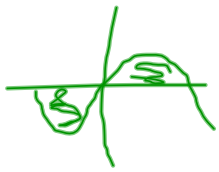
45) f; ; is odd $\cdot$ $[f(-x) = -f(x)]$

$$\int_{-a}^{a} f(x)\,dx = ?$$

$$\int_{-a}^{a} f(x)\,dx = \int_{-a}^{0} f(x)\,dx + \int_{0}^{a} f(x)\,dx$$

$$= \int_{0}^{a} -f(x)\,dx + \int_{0}^{a} f(x)\,dx$$

$$= \int_{0}^{a} 0\,dx = 0$$



$$= -\int_{0}^{a} f(x)\,dx + \int_{0}^{a} f(x)\,dx = 0$$

(b) $\int_{-1}^{1} x^3\,dx$ and $\int_{-\pi/2}^{\pi/2} \sin x\,dx$

(c) Q. $\int_{-a}^{0} f(x)\,dx$ w/ $\int_{0}^{a} f(x)\,dx$; where $f(x)$ even $[f(-x) = f(x)]$

$$\int_{-a}^{0} f(x)\,dx = \int_{0}^{a} f(x)\,dx$$

53) Let $F(x)=\int_2^x \sqrt{3t^2+1}\,dt$

a) $F(2)=\int_2^2 \sqrt{3t^2+1}\,dt=0$

b) $F'(2)=?$

$\frac{d}{dx}\left(\int_2^x \sqrt{3t^2+1}\,dt\right)=\sqrt{3x^2+1}$

$F'(2)=\sqrt{3(2)^2+1}=\sqrt{13}$

c) $F''(2)=?$

$\frac{d}{dx}\left(\sqrt{3x^2+1}\right)=\frac{d}{dx}(3x^2+1)^{1/2}=\frac{1}{2}(3x^2+1)^{-1/2}(6x)$

$F''(x)=\frac{3x}{\sqrt{3x^2+1}}$

$F''(2)=\frac{6}{\sqrt{13}}$

d) $F(x)=\int_x^2 \sqrt{3t^2+1}\,dt$

$F'(x)=?$ $\frac{d}{dx}\left(\int_x^2 \sqrt{3t^2+1}\,dt\right)=\frac{d}{dx}\left(-\int_2^x \sqrt{3t^2+1}\,dt\right)$

$=-\sqrt{3x^2+1}$.

14) $\int_1^8 \left(5x^{2/3} - 4x^{-2}\right) dx$

step 1 } find an antiderivative; i.e. $\int 5x^{2/3} - 4x^{-2}\, dx$

$$\int 5x^{2/3} - 4x^{-2}\, dx = 5\frac{x^{5/3}}{5/3} - 4\frac{x^{-1}}{-1} + C$$

$$= 3x^{5/3} + \frac{4}{x} + C$$

step 2 } evaluate $\int_1^8 5x^{2/3} - 4x^{-2}\, dx =$

$$\left(3x^{5/3} + \frac{4}{x}\right)\Big|_1^8 = \left[3(8)^{5/3} + \frac{4}{8}\right] - \left[3+4\right]$$

$$= \left[3\left(8^{1/3}\right)^5 + \frac{1}{2}\right] - \left[7\right] = \left[3\cdot 32 + \frac{1}{2}\right] - 7$$

$$= 89\tfrac{1}{2} = \frac{179}{2}$$

2 FTC $\int_a^b f(x)\, dx = F(b) - F(a)$

where $F(x) = \int f(x)\, dx$

6.6 homework

2010-11-29 Pd 3

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